Problem 1

The domain of arcos(x) is −1 ≤ *x* ≤ 1 ,  
the range of arcos(x) is [0 , π] ,  
 arcos(x) is the angle in [0, π] whose cosine is x.

Properties of the function y=arccos(x)y=arccos⁡(x):

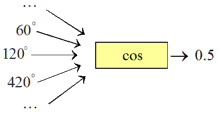
1. Domain is segment [-1,1].
2. Range is segment [0,π][0,π].
3. Function is neither even, nor odd.
4. Function is decreasing.

**The arccosine function**

**Background:** The arccosine function is the **inverse** of the [cosine](http://mathonweb.com/help_ebook/html/functions_1.htm) function (as long as the cosine function is restricted to a certain domain). Click here for a review of [inverse functions](http://mathonweb.com/help_ebook/html/functions_6.htm#inverse).  
  
Recall that the cosine function takes an angle *x* as input and returns the cosine of that angle as output:

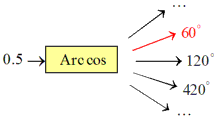
http://mathonweb.com/help_ebook/html/functions/functions43a.gif

For example if 60° is the input then 0.5 is the output. Here we want to create the inverse function that would take 0.5 as input and return 60° as output. But there is a problem. Notice that there are many angles whose cosine is 0.5:



We say that this mapping is **many-to-one**. This means that the inverse mapping would be one-to-many and therefore would not satisfy the [‘one range value’ requirement](http://mathonweb.com/help_ebook/html/functions_6.htm) for a mapping to be a function. To fix this problem we introduce both a [**relation**](http://mathonweb.com/help_ebook/html/functions_6.htm#vertical) called Arccosine (with capital A and abbreviation Arccos) and a **function** called arccosine (with lower case ‘a’ and abbreviation arccos). Here is how they are defined:

**Definition:** The Arccosine of *x*, denoted Arccos(*x*), is defined as ‘**the set of all angles** whose cosine is *x*’. It is a one-to-many **relation**. Here is an example:



**Definition:** The arccosine of *x*, denoted arccos(*x*), is defined as ‘the **one angle** between 0 and π radians (or between 0° and 180°) whose cosine is *x*’. It is a one-to-one **function**. Here is an example:

http://mathonweb.com/help_ebook/html/functions/functions46a.gif

**Definition:** Of all the values returned by the Arccosine relation, the one that is the same as the value returned by the arccosine function is called the **principal value** of the Arccosine relation. (An example is the value 60° shown above in red.)